SOFT HANDOFF MODELING IN CDMA CELLULAR SYSTEMS

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Abstract— **We propose a new analytical soft handoff model in a rectangular cell configuration and obtain the probability distribution of the sojourn time in the soft handoff region. The movements of a mobile user are categorized into several transition cases for calculating the probability distribution. We obtain the probability distribution for each transition case and the resulting probability distribution by unconditioning the transition cases. We can derive various probability distributions by varying model parameters and observe the effects of parameters. These distributions can be utilized in characterizing various traffic characteristics such as soft handoff traffic and mean handoff rate.**

I. Introduction

The same frequency band can be used simultaneously over neighboring cells in CDMA cellular systems. This enables a mobile to keep its frequency band when a handoff occurs, since it usually supports only one radio frequency channel. Then the mobile can maintain several connections from two or three base stations. In this handoff scheme, a new base station modem is assigned to a mobile while the old base station continues to serve the call. The handoff cannot be recognized to the user, since the mobile is always connected to at least one base station. This handoff scheme called the soft handoff can make a seamless call connection. The old base station releases the call, and the only newly assigned base station serves the call if the received signal strength is below and above the given thresholds in the new cell, respectively.

It is experimentally known that the proportion of soft handoff calls is about 30%. However, the soft handoff phenomena have not been fully characterized yet, because they depend on various parameters such as user mobility behaviors. Some simulation studies [1] have been done instead of analytical approaches because of the complexity of modeling. We analytically modeled the soft handoff by using a dual concentric circular cell in our previous work [2]. However, the complex boundary conditions required some approximations.

In order to characterize this soft handoff mechanism, we propose a new CDMA soft handoff model by simplifying the boundary conditions of our previous work [2] in this study, and then derive *the probability distribution of the sojourn time in the soft handoff region*, in which the sojourn time means the time duration of occupying both channels between a mobile and its two neighboring base stations until a call is completed or it is completely handed off to a new neighboring cell.

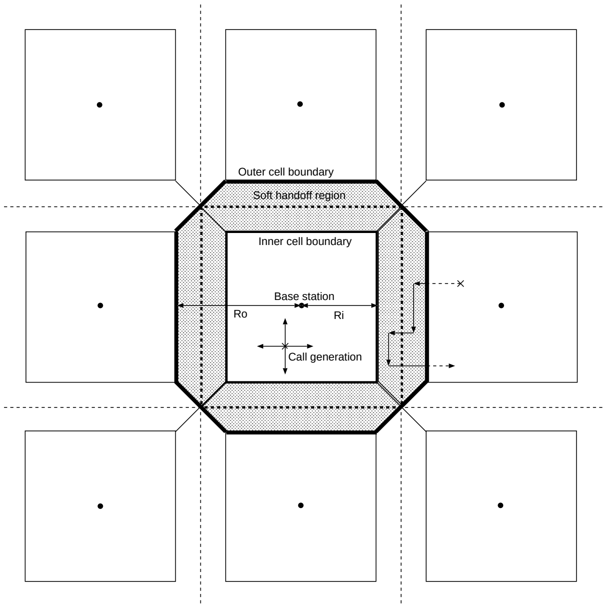

Fig. 1. Cell model

This paper is organized as follows. In section II, we model user mobility in a given cell configuration. This modeling includes various assumptions and several random variables. We obtain the probability distribution of the sojourn time in the soft handoff region. In section III, some analytical results on various cases are given. Finally, conclusions are drawn in section IV.

II. Soft Handoff Modeling

A. Modeling assumptions

In order to model the soft handoff, we have the following assumptions:

- Calls are uniformly generated in a cell.
- A base station is located at the origin of the Cartesian coordinates.
- The antenna of a base station is omni-directional.
- Only the relative path loss is considered, i.e., fast fading and shadowing effect are not considered here because they do not affect the average power level.
- A cell is the interior region surrounded by a dot-lined rectangle centered at the origin as shown in Fig.1. For computational convenience, we consider a rectangular cell configuration.

- Soft handoff region is surrounded by inner cell and outer cell boundaries as shown in Fig.1. A soft handoff call occurs when a mobile station either crosses over the inner cell or outer cell boundary, or makes a call inside the soft handoff region.
- A mobile can move in right, left, up, or down directions.
- At the point of call generation, the mobile selects one of four directions with identical probability $\frac{1}{4}$.
- The speed of a mobile V keeps constant until the next change of velocity or the completion of a call.
- V is uniformly distributed over $[0,\ V_{max}]$. The two speeds of a mobile before and after the change of velocity are independent and identically distributed.
- V and T, the time between consecutive changes of velocity are statistically independent.
- The number of velocity changes during a call, M is geometrically distributed. Its memoryless property makes the call holding time follow the exponential distribution.

B. Probability distribution of the sojourn time

From the above assumptions, we introduce random variables and obtain the required probabilities, probability density functions, and cumulative distribution functions. The probability density function of V is given by

$$f_V(v) = \frac{1}{V_{max}}\ (0 \le v \le V_{max})\ . \tag{1}$$

The probability density function of T is expressed as

$$f_T(t) = \nu e^{-\nu t}\ (t > 0)\ , \tag{2}$$

where $\frac{1}{\nu}$ is the mean time for one straight movement. And the geometrically distributed probability mass function of M is written as

$$Prob(M = m) = (1-p)p^m\ , \tag{3}$$

where p is the probability that the mobile executes another straight movement. From the above density functions, the call duration time Γ is exponentially distributed with parameter (1-p)ν. The travelling distance S is defined as the product of V and T.

$$\begin{aligned} S &= V \times T\ . & (4)\\ f_S(s) &= \frac{\nu}{V_{max}} \int_{\frac{\nu}{V_{max}}s}^{\infty} \frac{1}{u} e^{-u} du\ . & (5) \end{aligned}$$

We divide the cell area into three regions, A, B, and C as shown in Fig.2 to calculate several conditional probabilities. Fig.2 shows all possible transition cases caused by one straight movement. For example, the transition from A to B means that a mobile user moves from region A to region B. TABLE I shows 18 possible transition cases.

First, $Prob(A)$ denotes the probability that new calls are generated in region A. Then from the ratio of areas

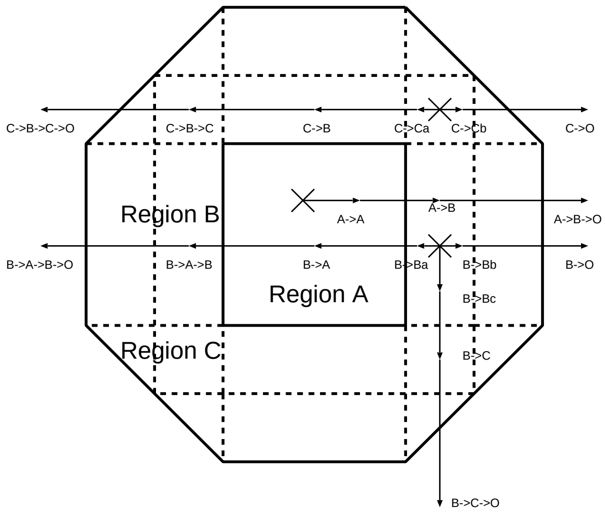

Fig. 2. Several transition cases

TABLE I
POSSIBLE TRANSITIONS

Number	Transition cases
1	$A \to A$
2	$A \to B$
3	$A \to B \to O$
4	$B \to B_a$
5	$B \to B_b$
6	$B \to B_c$
7	$B \to A$
8	$B \to C$
9	$B \to O$
10	$B \to A \to B$
11	$B \to A \to B \to O$
12	$B \to C \to O$
13	$C \to C_a$
14	$C \to C_b$
15	$C \to O$
16	$C \to B$
17	$C \to B \to C$
18	$C \to B \to C \to O$

$Prob(A)$, $Prob(B)$, and $Prob(C)$ are given by

$$\begin{aligned} Prob(A) &= \left(\frac{R_i}{R_{eq}}\right)^2 & (6)\\ Prob(C) &= \left(\frac{R_{eq}-R_i}{R_{eq}}\right)^2 \\ Prob(B) &= 1 - Prob(A) - Prob(C)\ , \end{aligned}$$

where $R_{eq} = \frac{R_i+R_o}{2}$. And all the transition probabilities conditioned by their call originating regions are given in Appendix.

For each transition case, we compute the cumulative distribution functions of the sojourn time in the soft handoff region. W_n denotes the sojourn time in the soft handoff region for the n-th transition case. And $F_{W_n}(t)$ is the cumulative distribution function of the sojourn time in the soft handoff region for the n-th transition case. W_1 is the sojourn time in the case of transition from A to A(See Fig.2). Since there is no soft handoff region in this case, W_1 takes only zero.

$$P(W_1 = 0) \;=\; 1\;. \tag{7}$$
$$F_{W_1}(t) \;=\; u(t)\;. \qquad :\ \text{unit step function} \tag{8}$$

And other cumulative distribution functions of the sojourn time in the soft handoff region are given in Appendix.

Let n_i and $W_{i,j}$ denote the number of occurrences of the i-th transition case and the sojourn time in the j-th occurrence ($j \le n_i$) of transition case i, respectively. We here note that $F_{W_{i,1}} = F_{W_{i,2}} = \cdots = F_{W_{i,n_i}} = F_{W_i}$. The total sojourn time in the soft handoff region is denoted by U, and then the distribution of U under the condition of m velocity changes is given by

$$Prob(U \le t \mid M = m) = \sum_{n_1+\cdots+n_{18}=m+1} Prob(U \le t \mid (n_1, ..., n_{18}; m))Prob(n_1, ..., n_{18}; m)\;, \tag{9}$$

where

$$Prob(U \le t \mid (n_1, ..., n_{18}; m)) = F_{W_{2,1}} * \cdots * F_{W_{2,n_2}} * F_{W_{3,1}} * \cdots * F_{W_{3,n_3}} * \cdots * F_{W_{18,n_{18}}}(t)\;.$$

We can obtain the cumulative distribution function of the total sojourn time in the soft handoff region by unconditioning M.

$$Prob(U \le t) = \sum_{m=0}^{\infty} Prob(U \le t \mid M = m)(1-p)p^m\;. \tag{10}$$

Since it is difficult to condition $(n_1, ..., n_{18}; m)$ for $m > 0$, we approximate the above equations as follows.

$$Prob(U \le t \mid M = m) = Prob(U \le t \mid M = 0)^{(n)*}\;. \tag{11}$$

The righthand side of (11) implies an n-fold convolution.

C. Validation

Fig.3 illustrates a comparison between the analytical result and the simulation one in the case of $\zeta = 0.75$, $V = 30\ km/h$, $\frac{1}{\nu} = 1\ min.$, $M = 0$, and $R_{eq} = 1\ km$. The result shows that the analytical result agrees well with the simulation one.

III. Numerical Examples

We now obtain the resulting probability distribution of the sojourn time in the soft handoff region by specifying the following parameters: the speed of a mobile V, the parameter ν, the parameter p, the cell size R_{eq}, and the area ratio between inner cell region and cell region ζ. For $\frac{1}{\nu} = 1\ min.$, $p = 0.5$, and $R_{eq} = 1\ km$, we apply various values of V and ζ. And M is restricted to 0, 1, and 2 for computational simplicity.

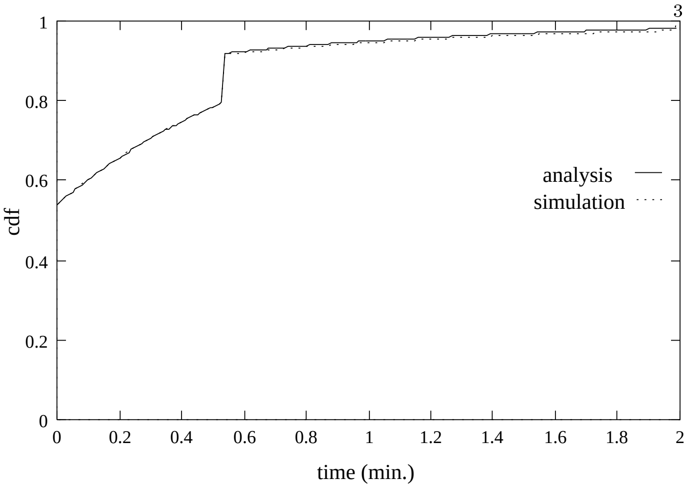

Fig. 3. Comparison between the analytical and the simulation results ($\zeta = 0.75$, $V = 30\ km/h$, $\frac{1}{\nu} = 1\ min.$, $M = 0$, $R_{eq} = 1\ km$)

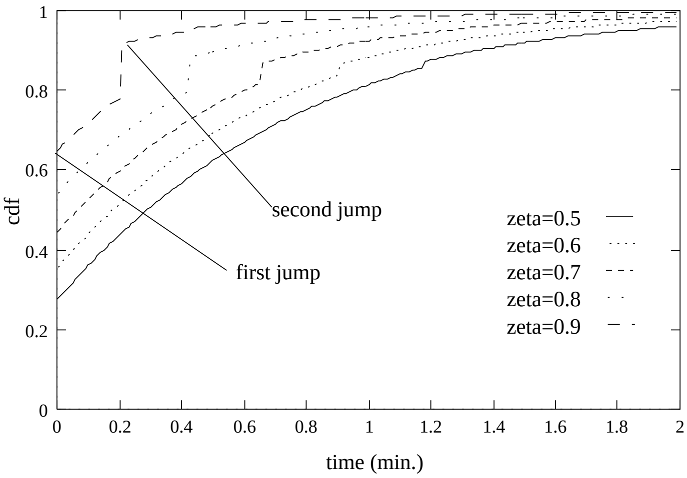

Fig. 4. Probability distributions of the sojourn time in the soft handoff region ($V = 30\ km/h$)

We obtain the probability distributions of the sojourn time in the soft handoff region for varying the parameter ζ in the case of $V = 30\ km/h$. The result is shown in Fig.4. Each graph has two jumps: the first one is located at zero sojourn time in the soft handoff region, and the second one is due to calls which pass through the soft handoff region only once. Let us take an example for an $A \to B \to O$ call. As ζ increases, the heights of jumps increase. This can be explained as follows. As ζ increases, inner cell area increases and non-soft-handoff calls increase. As the soft handoff region decreases, the probability increases that a call passes through the soft handoff region.

Fig.5 shows the probability distribution of the sojourn time in the soft handoff region for varying user velocity. High velocity causes soft handoff calls to occur more frequently and to pass shortly through the soft handoff region. These effects are illustrated in Fig.5 as the smaller first jump and the bigger second one in the case of higher velocity.

Even though we here show cumulative distribution func-

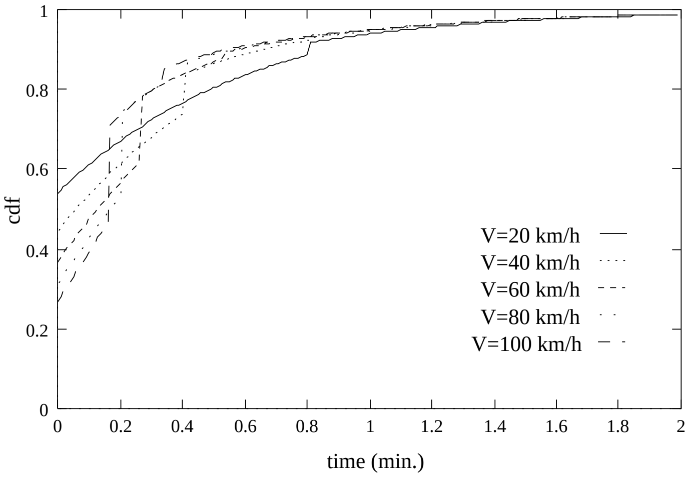

Fig. 5. Probability distributions of the sojourn time in the soft handoff region ($\zeta = 0.75$)

tions, we can obtain the first moment, the second moment, and variance from the above distributions.

IV. Conclusion

We proposed a new analytical soft handoff model in a rectangular cell configuration, and derived the probability distributions of the sojourn time in the soft handoff region. They depend on the parameters such as the speed of a mobile, the call duration time, the cell size, and the ratio of the soft handoff region area. Varying the above parameters, we can obtain various resulting probability distributions. These results can be utilized in characterizing various traffic characteristics such as soft handoff traffic and mean handoff rate.

Appendix

The transition probabilities conditioned by their call originating regions are given as follows.

$$\begin{aligned}
P_1 &= Prob\{A \rightarrow A\} = \int_0^{2R_i} Prob\{S < z\} \frac{1}{2R_i} dz \\
&= \frac{\nu}{V_{max}} \frac{1}{2R_i} \int_0^{2R_i} \int_0^{z} \int_{\frac{\nu}{V_{max}} s}^{\infty} \frac{1}{t} e^{-t} dt\ ds\ dz \\
P_2 &= Prob\{A \rightarrow B\} \\
&= \frac{\nu}{V_{max}} \frac{1}{2R_i} \int_0^{2R_i} \int_z^{z+R_o-R_i} \int_{\frac{\nu}{V_{max}} s}^{\infty} \frac{1}{t} e^{-t} dt\ ds\ dz \\
P_3 &= Prob\{A \rightarrow B \rightarrow O\} \\
&= \frac{\nu}{V_{max}} \frac{1}{2R_i} \int_0^{2R_i} \int_{z+R_o-R_i}^{\infty} \int_{\frac{\nu}{V_{max}} s}^{\infty} \frac{1}{t} e^{-t} dt\ ds\ dz \\
P_4 &= Prob\{B \rightarrow B_a\} \\
&= \frac{1}{4} \frac{\nu}{V_{max}} \frac{1}{R_{eq} - R_i} \int_0^{R_{eq}-R_i} \int_0^{z} \int_{\frac{\nu}{V_{max}} s}^{\infty} \frac{1}{t} e^{-t} dt\ ds\ dz
\end{aligned}$$

Similarly, the remaining transition probabilities, P_5 to P_{18} can be obtained.

The cumulative distribution functions of the sojourn time in the soft handoff region are given as follows.

$$F_{W_2}(t) =$$

$$\frac{1}{V_{max}} \int_0^{V_{max}} \frac{\int_0^{2R_i} Prob(x < S \leq min(x + R_O - R_I, x + vt))dx}{\int_0^{2R_i} Prob(x < S \leq x + R_O - R_I)dx} dv$$

$$F_{W_3}(t) = \frac{1}{V_{max}} \int_0^{V_{max}} u(vt - R_o + R_i) dv$$

$$F_{W_4}(t) = \frac{1}{V_{max}} \int_0^{V_{max}} \frac{\int_0^{R_{eq}-R_i} Prob(S \leq min(x, vt))dx}{\int_0^{R_{eq}-R_i} Prob(S \leq x)dx} dv$$

Similarly, the remaining cumulative distribution functions, F_{W_5} to $F_{W_{18}}$ can be obtained.

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