Comments on "A Class of Fault-Tolerant Multiprocessor Networks"

Jong-Seok Kim, Hyeong-Ok Lee, and Sung Won Kim

***Abstract*—A. Ghafoor presented node-disjoint paths of even networks using Figs. 4, 5, 6,and 7 (Ghafoor, IEEE Trans. Reliability, vol. 38, no. 1, pp. 5–15). However, the paper contains errors which cause confusion. We show that the node-disjoint paths, and Theorem 4 (Ghafoor, IEEE Trans. Reliability, vol. 38, no. 1, pp. 5–15), are not correct. We propose advanced node-disjoint paths, and prove that the fault diameter of even networks is $d+1$. This is optimal.**

***Index Terms*—Even networks, fault diameter, interconnection networks, node-disjoint paths.**

NOTATION

G an interconnection network

$k(G)$ the connectivity of G

E_d an even network

d the degree of E_d

$|l|$ the number of l

x, y arbitrary nodes in E_d

$\bar{x}$ complementary node of node x

H_{xy} the Hamming distance between two binary codewords, x and y, the number of positions at which these codewords differ

L_{xy} the graphical distance between two nodes, x and y, $\min\{H_{xy}, 2d-2-H_{xy}\}$

β an edge connecting two nodes x and y, where H_{xy} is $2d-3$

S_{ij}^{xy} the set of positions in the codewords associated with nodes x and y, such that if x has bit value i, then y has bit value j $(i, j = 0, 1)$

α_i an operator which, when it operates on a codeword x, yields the codeword y, with which x has the ith bit $(=1)$ complemented

α_t^{ij} an operator α_t with $t \in S_{ij}^{xy}$

Λ_1 a path A_1 in [5]

A_n the shortest path between x, and y in Fig. 1, when $L_{xy} = H_{xy} = \text{even}, 1 \le n \le a, a = L_{xy}/2$

B_n the shortest path between x, and y in Fig. 2, when $L_{xy} = H_{xy} = \text{odd}, 1 \le n \le b, b = (L_{xy}+1)/2$

C_n the shortest path between x, and y in Fig. 3, when $L_{xy} \ne H_{xy} = \text{even}, 1 \le n \le c, c = (L_{xy}/2)+1$

F_n The shortest path between x, and y in Fig 4, when $L_{xy} \ne H_{xy} = \text{odd}, 1 \le n \le f, f = (L_{xy}-1)/2$

G_m an alternate path between x, and y in Fig. 5, when $L_{xy} = H_{xy} = \text{even}, 1 \le m \le g, g = d-a-1$

H_m an alternate path between x, and y in Fig 6, when $L_{xy} = H_{xy} = \text{odd}, 1 \le m \le h, h = d-b-1$

Z_m an alternate path between x, and y in Fig. 7, when $L_{xy} \ne H_{xy} = \text{even}, 1 \le z \le a, z = d-c$

W_m an alternate path between x, and y in Fig. 8, when $L_{xy} \ne H_{xy} = \text{odd}, 1 \le w \le a, w = d-f-1$

P a path of A_1 in reverse order in Fig. 1

P' a path of F_1 in reverse order in Fig. 4

Manuscript received November 16, 2007; revised August 05, 2008 and October 14, 2008; accepted October 18, 2008. First published May 27, 2009; current version published September 02, 2009. Associate Editor: J. Rupe.

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Digital Object Identifier 10.1109/TR.2009.2020101

I. INTRODUCTION

In massive multicomputer systems, the interconnection network plays a crucial role in issues such as communication performance, hardware cost, potentialities for efficient applications, and fault tolerance capabilities. The concept of node-disjoint paths arose naturally from the study of routing, reliability, fault tolerance, and communication protocols in multicomputer systems. A set of paths is said to be *node-disjoint* if no node except the source node and the destination node appears in more than one path. In this paper, node-disjoint paths are composed of the shortest paths, and alternate paths. A *path* is a sequence of connected nodes. The *shortest path* is a path of length L_{xy} between x and y in node-disjoint paths; and the *alternate path* is any path among paths that are node-disjoint paths, but not the shortest path. For a node x, we denote by $\alpha_1 - \alpha_2 - \ldots - \alpha_t$ a path obtained by applying operators $\alpha_1, \alpha_2, \ldots, \alpha_t$ to x. It is important to have node-disjoint paths between any two nodes in an interconnection network to speed up the transfer of large amounts of data, and provide alternative routes in cases of node, and/or link failures. Therefore, it is important that each node-disjoint path operates correctly. A common notion of fault tolerance in interconnection networks is based on the connectivity of the network. *Fault-tolerance* is the property that enables a network to continue operating properly in the event of the failure of (or one or more faults within) some of its components. The *connectivity (or node-connectivity)* of G is the smallest number of nodes whose removal disconnects G. In an interconnection network with connectivity of $k(G)$, the network is guaranteed to remain connected even if $k(G)-1$ node processors fail. However, while the connectivity of such a network is still preserved, the network diameter may increase significantly. The *diameter* of G is the distance between the two nodes which are furthest from each other. A good measure to judge this fault tolerance aspect of the network is fault diameter. The concept of fault diameter was first proposed by Krishnamoorthy & Krishnamurthy [8]. The *fault diameter* of G is the maximum length of the shortest paths between all two fault-free nodes when there are $k(G)-1$ or less faulty nodes. The fault diameters of many well-known networks have been determined by several researchers [1]–[4], [7], [8], [10], [11]. In [5], Ghafoor introduced even network E_d

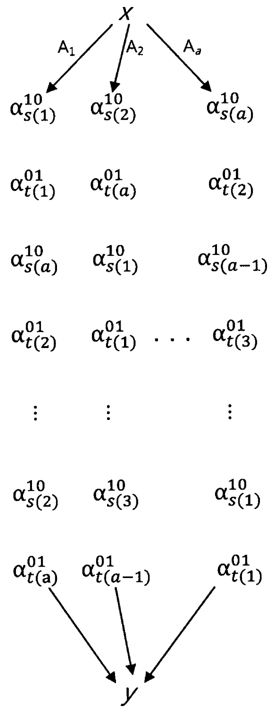

Fig. 1. The shortest paths, when $L_{xy} = H_{xy} = \mathrm{even}, a = L_{xy}/2$.

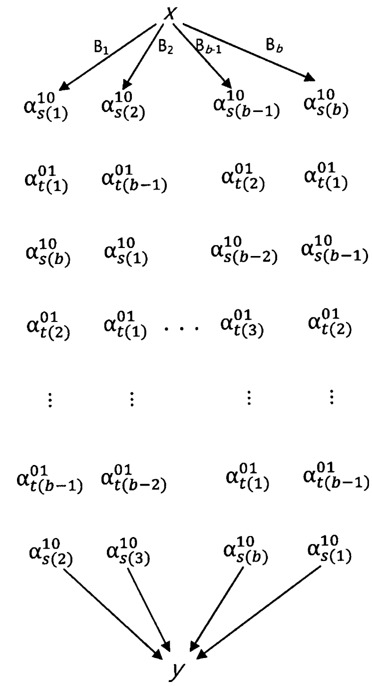

Fig. 2. The shortest paths, when $L_{xy} = H_{xy} = \mathrm{odd}, b = (L_{xy} + 1)/2$.

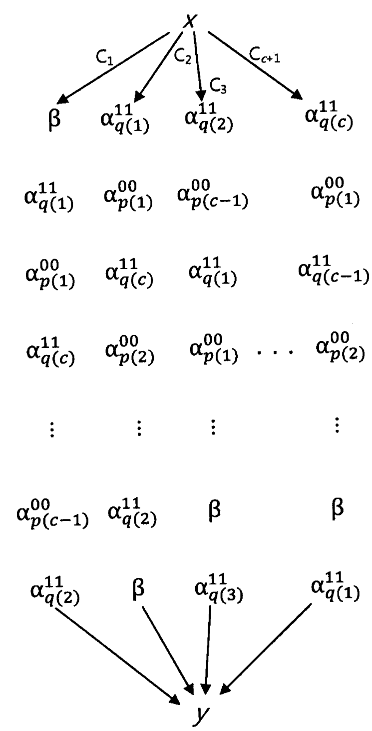

Fig. 3. The shortest paths, when $L_{xy} \neq H_{xy} = \mathrm{even}, c = (L_{xy}/2) + 1$.

to model fault-tolerant multiprocessor networks. *Even networks* are interconnection networks such that each node has the same number of edges, d, and the number of nodes is $\binom{2d-2}{d-1}$. E_d with $d \geq 2$ has the set of binary codewords of length $2d - 3$ with $|1| = |0| \pm 1$ as its node set. The degree of E_d is d, and the diameter of E_d is $d - 1$. The *degree* of x is the number of edges meeting at x. Two nodes are adjacent iff the Hamming distance between the two nodes is 1 or $2d - 3$. Several important properties including node-disjoint paths of E_d have been analyzed [5], [6], [9]. By introducing node-disjoint paths, the fault diameter of E_d can be $d + 2$ $(d = \mathrm{odd})$, or $d + 3$ $(d = \mathrm{even})$. In this comment, we show that the node-disjoint paths, and Theorem 4 proposed in [5], are incorrect. We propose a correct version of node-disjoint paths, and prove that the fault diameter of even networks is $d + 1$. This result is optimal, and will result in more accurate, safer information delivery in E_d.

II. Errors of Node-Disjoint Paths in [5]

Ghafoor proposed node-disjoint paths of E_d using Figs. 4, 5, 6, 7 in [5], and proved the length of node-disjoint paths as follows [5].

Theorem 4 of [5]: The number of node-disjoint paths between any two nodes $x, y \in E_d$ is the maximum possible, and is equal to d. The lengths of such paths are

Case a) L_{xy} is even: There are $L_{xy}/2$ paths of length L_{xy}. The remaining paths are of equal length, which is $L_{xy} + 2$.

Case b) L_{xy} is odd: There are $(L_{xy} + 1)/2$ paths of length L_{xy}. There is one alternate path of length $L_{xy} + 2$. The remaining paths are of equal length, which is $L_{xy} + 4$.

There are several errors related to this theorem.

1) $\beta - \Lambda_1$ (in reverse order)-β, in Fig. 5 in [5] is incorrect. Some nodes included in this path are not the nodes of E_d. Note that the number of codewords that compose the nodes is $|1| \neq |0| \pm 1$. The author showed an example of node-disjoint paths in Fig. 3 in [5]. The path, $\beta - \Lambda_1$ (in reverse order)-β, on the example in Fig. 3 in [5] is $x = 00000111111 - 11111000000$ (by β) -11101000000 (by α_7) $-$ 11101100000 (by α_5) -11100100000 (by α_6) $-$ 11100110000 (by α_4) -00011001111 (by β) $= y$. However, the two nodes 11101000000 (by α_7), 11100100000 (by α_6) in this path are not the nodes of E_d because the number of codewords that compose those two nodes is $|0| = |1| + 3$.

2) Common nodes exist on $\bar{\Lambda}_1$ in Fig. 6 in [5], and the path, $\beta - \bar{\Lambda}_1 - \beta$ in Fig. 7 in [5].

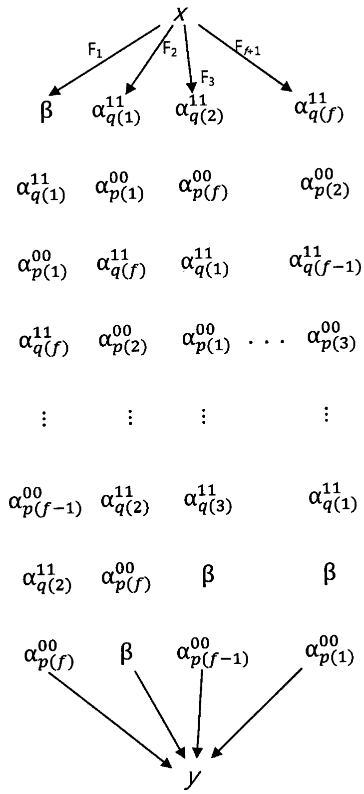

Fig. 4. The shortest paths, when $L_{xy} \neq H_{xy} = \mathrm{odd}, f = (L_{xy} - 1)/2$.

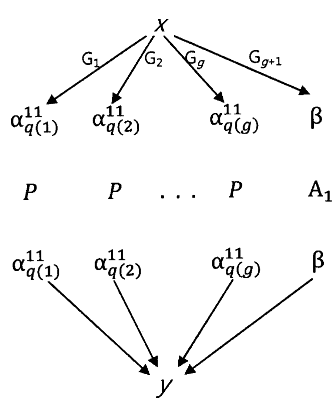

Fig. 5. Alternate paths, when $L_{xy} = H_{xy} = \mathrm{even}, g = d - 1 - a$.

Let $x = 0000001111111$, and $y = 1111100000001$. Then, $\bar{\Lambda}_1$: 0000001111111-0000001111110-1111110000001-1111100000001, $\beta - \bar{\Lambda}_1 - \beta$: 0000001111111-1111110000000-1111110000001-0000001111110-0000011111110-1111100000001. Common nodes are 0000001111110, 1111110000001. Therefore, $\bar{\Lambda}_1$, and $\beta - \bar{\Lambda}_1 - \beta$ are not node-disjoint paths.

3) Some of the figures do not clearly present all node-disjoint paths. Fig. 4 in [5] shows paths for $L_{xy} = H_{xy} = \mathrm{even}$ when we remove the sentence "replace with β, $L_{xy} \neq H_{xy}$". Fig. 5 in [5] shows alternate paths for $L_{xy} = H_{xy} = \mathrm{even}$, but it is in error (see the first item). Figs. 6, and 7 in [5] have the error given in our second item.

4) Theorem 4 in [5] is incorrect.
   Case i) Let $x = 0000001111111$, and $y = 1111111000000$. Then $L_{xy} = 2\ (= \mathrm{even})$. By case a) of Theorem 4, there is 1 path of length 2, when $L_{xy} = 2$. However, there are two paths of length $L_{xy} = 2$: 0000001111111-1111110000000-1111111000000, 0000001111111-0000000111111-1111111000000.
   Case ii) Let $x = 0000001111111$, and $y = 1111110000001$. Then $L_{xy} = 1\ (= \mathrm{odd})$. By case b) of Theorem 4, there are 1 path of length 1, 1 path of length 3, and 5 paths of length 5. However, there is no path of length 3, because the path $\beta - \beta - \beta$ in Fig. 7 in [5] does not exist.

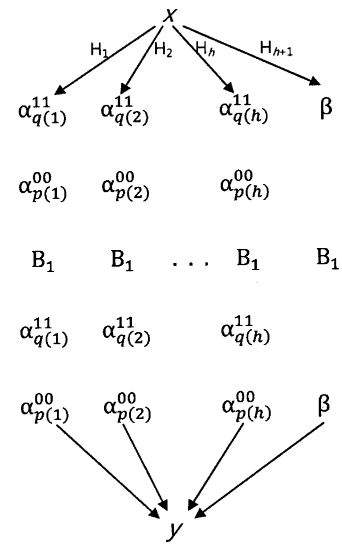

Fig. 6. Alternate paths, when $L_{xy} = H_{xy} = \mathrm{odd}, h = d - 1 - b$.

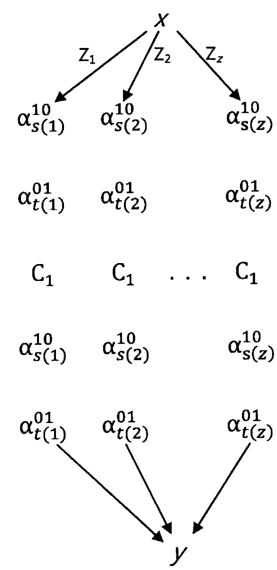

Fig. 7. Alternate paths, when $L_{xy} \neq H_{xy} = \mathrm{even}, z = d - c$.

III. Advanced Node-Disjoint Paths and Fault Diameter

We propose advanced node-disjoint paths, and prove that the fault diameter of E_d is $d + 1$. Even networks possess numerous symmetry properties including node, and edge symmetry [5]. G is said to be *node-symmetric* if, for any two nodes x, and y, there exists an automorphism of G that maps x into y. In other words, G has the same shape as viewed from any node. We write a node $\overbrace{0\ldots0}^{d-2}\overbrace{1\ldots1}^{d-1}$ in E_d as $0^{d-2}1^{d-1}$. Because E_d is node-symmetric,

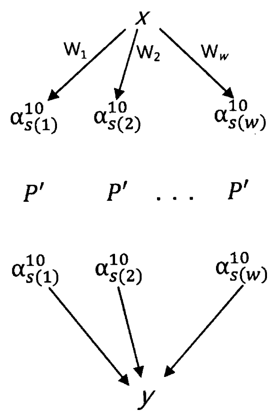

Fig. 8. Alternate paths, when $L_{xy} \neq H_{xy} = \mathrm{odd}$, $w = d - f - 1$.

$x = 0^{d-2}1^{d-1}$. Advanced node-disjoint paths of E_d $(d \geq 3)$ are shown in Fig. 1 through Fig. 8.

We show that the paths we proposed are node-disjoint using Lemmas 1, 2, and 3.

Lemma 1: All of the paths A_n $(1 \leq n \leq a)$ in Fig. 1 are node-disjoint.

Proof: Because E_d is node-symmetric, let two given nodes be $x = 0^{d-2}1^{d-1}$, and y. As shown in Fig. 1, these paths are permuted sequences of the operators α_i from x to y $(1 \leq i \leq 2a)$. In these paths, operators of the same type, say $\alpha^{10}_{s(i)}$, appear at the odd levels, while the others appear at the even levels. These paths are of the shortest possible length, because the selection of the operators α_i in each path is consistent with the shortest path routing algorithm. Consider two paths in Fig. 1, say A_1, and A_i, where A_i is some ith cyclically permuted version of A_1. Suppose there is a common node w $(\neq x, y)$ in two paths. Then, the selection of the operators α_i from x to w in two paths must be the same. However, this is impossible because A_i is some ith cyclically permuted version of A_1. Therefore, there is no common node w $(\neq x, y)$ in the two paths, and both A_1, and A_i are node-disjoint. Similarly, it can be proven that all of the paths in Figs. 2, 3, and 4 are node-disjoint. ■

Lemma 2: All of the paths G_m $(1 \leq m \leq g)$ in Fig. 5 are node-disjoint. In addition, G_m, and G_{g+1} are node-disjoint.

Proof: Because $\alpha^{11}_{q(i)}$ $(1 \leq i \leq g)$, and β are unique, all of the paths in Fig. 5 are node-disjoint. Similarly, all of the paths in Figs. 6, 7, and 8 are node-disjoint. ■

Lemma 3: A_n $(1 \leq n \leq a)$, and G_m $(1 \leq m \leq g+1)$ are node-disjoint.

Proof: Let $\alpha^{11}_{q(i)} - P$ be a path from x to y' in E_d. y' is a neighbor node of y by $\alpha^{11}_{q(i)}$. Then, $\alpha^{11}_{q(i)} - P$, and $A_l - \alpha^{11}_{q(i)}$ are node-disjoint by Lemma 1. Connecting $\alpha^{11}_{q(i)} - P$, and $A_l - \alpha^{11}_{q(i)}$ constitutes a cycle. Therefore, A_l, and $\alpha^{11}_{q(i)} - P - \alpha^{11}_{q(i)}$ are node-disjoint. Also, A_l, and $\beta - A_1 - \beta$ are node-disjoint. Similarly, we can know that all paths in Fig. 2 and Fig. 6, Fig. 3 and Fig. 7, Fig. 4 and Fig. 8 are node-disjoint within their paired sets. ■

Lemma 4: An arbitrary sequence of distinct operators α_i $(1 \leq i \leq 2d-3)$ is joined to β in E_d, and constitutes a cycle.

Proof: Let u be an arbitrary node in E_d. An arbitrary node u is connected to its complementary node $\bar{u}$ by a path proposed of distinct operators α_i $(1 \leq i \leq 2d-3)$. And $\bar{u}$ is connected to u by β. Hence, the proof is completed. ■

We can easily check that the fault diameter of E_3 is 3. Therefore, we will prove the fault diameter of E_d, $d \geq 4$. The fault diameter of E_d we proposed is in the next theorem.

Theorem 1: The fault diameter of $E_d = d+1$ $(d \geq 4)$. This is optimal.

Proof: Because E_d is node-symmetric, let two given nodes be $x = 0^{d-2}1^{d-1}$, and y in E_d.

Case 1) $L_{xy} = \mathrm{even}$.

Case 1.1) $L_{xy} = d-1$: There are $L_{xy}/2$ paths of the form A_n, and $d - (L_{xy}/2)$ paths of the form C_m of length L_{xy} $(< d+1)$. A_n $(1 \leq n \leq a)$, and C_m $(1 \leq m \leq g+1)$ are node-disjoint by lemma 4. This means that the paths have optimal length.

Case 1.2) $L_{xy} = 2d-2-H_{xy} = d-2$: There are $L_{xy}/2$ paths of the form C_n, and $d - (L_{xy}/2)$ paths of the form A_m of length $L_{xy}+2$. C_n has optimal length. C_n $(1 \leq n \leq c+1)$, and A_m $(1 \leq m \leq a)$ are node-disjoint by lemma 4. E_d is a bipartite graph [9], so it cannot contain an odd cycle. By lemma 4, if C_n is joined to A_m, it constitutes a cycle. The length of A_m cannot be $L_{xy}+1$. Therefore, the length of A_m is $L_{xy}+2$ $(< d+1)$; this means that A_m has optimal length.

Case 1.3) $L_{xy} = 2d-2-H_{xy}$: There are $L_{xy}/2$ paths of length L_{xy}. The remaining paths are of equal length, which is $L_{xy}+4$ according to Fig. 3, and Fig. 7. C_n has optimal length; because the length of Z_m $(1 \leq m \leq z)$ is greater than the length of C_n, the length of Z_m is not L_{xy}. E_d is a bipartite graph, so it cannot contain an odd cycle. By Lemma 3, if C_n is joined to Z_m, it constitutes a cycle. The length of the alternate path cannot be $L_{xy}+1$, and $L_{xy}+3$. Suppose an alternate path is $\alpha^{10}_{s(m)} - C_1 - \alpha^{10}_{s(m)}$. Then, according to the proof of Lemma 3, the path $\alpha^{10}_{s(m)} - C_1$, and the path $C_1 - \alpha^{10}_{s(m)}$ from x shall lead to the same node. However, this is impossible, because it cannot use the operators $\alpha^{10}_{s(i)}$, β, and $\alpha^{11}_{q(1)}$ from x continuously. So, the length of Z_m is not $L_{xy}+2$. According to the proof of Lemma 3, the path $\alpha^{10}_{s(m)} - \alpha^{01}_{t(m)} - C_1$, and the path $C_1 - \alpha^{01}_{t(m)} - \alpha^{10}_{s(m)}$ from x lead to the same node. So, C_n, and Z_m from x lead to the same node y. Therefore, the length of Z_m is $L_{xy}+4$ $(\leq d+1)$; this means that Z_m has optimal length.

Case 1.4) $L_{xy} = H_{xy}$: There are $L_{xy}/2$ paths of length L_{xy}. The remaining paths are of equal length, which is $L_{xy}+2$ according to Fig. 1, and Fig. 5. A_n has optimal length; because the length of G_m $(1 \leq m \leq g+1)$ is greater than the length of A_n, the length of G_m is not L_{xy}. E_d is a bipartite graph, so it cannot contain an odd cycle. By Lemma 3, if A_n is joined to G_m, it constitutes a cycle. The length of G_m cannot be $L_{xy}+1$. Therefore, the length of G_m is $L_{xy}+2$ $(< d+1)$; this means that G_m has optimal length.

Case 2) $L_{xy} = \mathrm{odd}$.

Case 2.1) $L_{xy} = d-1$: There are $(L_{xy}+1)/2$ paths of the form B_n, and $d - ((L_{xy}+1)/2)$ paths of the form F_m of length L_{xy} $(< d+1)$. B_n $(1 \leq n \leq b)$, and F_m $(1 \leq m \leq f+1)$ are node-disjoint by Lemma 4. This means that the paths have optimal length.

Case 2.2) $L_{xy} = 2d - 2 - H_{xy} = d - 2$: There are $(L_{xy} + 1)/2$ paths of the form F_n, and $d - ((L_{xy} + 1)/2)$ paths of the form B_m of length $L_{xy} + 2$. F_n has optimal length. F_n $(1 \leq n \leq f+1)$, and B_m $(1 \leq m \leq b)$ are node-disjoint by Lemma 4. By Lemma 4, if F_n is joined to B_m, it constitutes a cycle. E_d is a bipartite graph, so it cannot contain an odd cycle. The length of B_m cannot be $L_{xy}+1$. Therefore, the length of B_m is $L_{xy}+2$ $(< d+1)$; this means that B_m has optimal length.

Case 2.3) $L_{xy} = H_{xy} = d - 2$: There are $(L_{xy} + 1)/2$ paths of the form B_n, and $d - ((L_{xy}+1)/2)$ paths of the form F_m of length $L_{xy} + 2$. B_n has optimal length. B_n $(1 \leq n \leq b)$, and F_m $(1 \leq m \leq f + 1)$ are node-disjoint by Lemma 4. By Lemma 4, if B_n is joined to F_m, it constitutes a cycle. E_d is a bipartite graph, so it cannot contain an odd cycle. The length of F_m cannot be $L_{xy}+1$. Therefore, the length of F_m is $L_{xy} + 2$ $(< d + 1)$; this means that F_m has optimal length.

Case 2.4) $L_{xy} = 2d - 2 - H_{xy}$: There are $(L_{xy} + 1)/2$ paths of length L_{xy}. The remaining paths are of equal length, which is $L_{xy}+2$ according to Fig. 4, and Fig. 8. F_n has optimal length; because the length of W_m $(1 \leq m \leq w)$ is greater than the length of F_n, the length of W_m is not L_{xy}. E_d is a bipartite graph, so it cannot contain an odd cycle. By Lemma 3, if F_n is joined to W_m, it constitutes a cycle. The length of the alternate path cannot be $L_{xy} + 1$. Therefore, the length of W_m is $L_{xy} + 2$ $(< d + 1)$; this means that the alternate path has optimal length.

Case 2.5) $L_{xy} = H_{xy}$: There are $(L_{xy} + 1)/2$ paths of length L_{xy}, and one alternate path, H_{h+1}, of length $L_{xy} + 2$. The remaining paths are of equal length, which is $L_{xy}+4$ according to Fig. 2, and Fig. 6. B_n has optimal length; because the length of H_m $(1 \leq m \leq h+1)$ is greater than the length of B_n, the length of H_m is not L_{xy}. E_d is a bipartite graph, so it cannot contain an odd cycle. By Lemma 3, if B_n is joined to H_m, it constitutes a cycle. The length of the alternate path cannot be $L_{xy} + 1$, and $L_{xy} + 3$. So, the length of H_{h+1} is $L_{xy} + 2$. It is optimal. Suppose an alternate path is $\alpha^{11}_{q(i)} - B_1 - \alpha^{11}_{q(i)}$ $(1 \leq i \leq h)$. Then, according to the proof of Lemma 3, the path $\alpha^{11}_{q(i)} - B_1$, and the path $B_1 - \alpha^{11}_{q(i)}$ from x shall lead to the same node. However, this is impossible, because it cannot use the operators $\alpha^{11}_{q(i)}$, and $\alpha^{10}_{s(1)}$ from x continuously. So, the length of H_i is not $L_{xy} + 2$. According to the proof of Lemma 3, the path $\alpha^{11}_{q(i)} - \alpha^{00}_{p(i)} - B_1$, and the path $B_1 - \alpha^{00}_{p(i)} - \alpha^{11}_{q(i)}$ from x lead to the same node. So, B_n, and H_i from x lead to the same node y. Therefore, the length of H_i is $L_{xy} + 4$ $(\leq d+1)$; this means that H_i has optimal length. ■

The fault diameter derived in this paper is better than the previously known bound.

ACKNOWLEDGMENT

The authors are grateful to the anonymous referees, and Dr. Jason W. Rupe for their helpful comments and suggestions.

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