(i) if $F_{OX} > F_{OXLIM}$, C_{Si} is essentially independent of N_{Si} and so is D_{SC}
(ii) if, instead, $F_{OX} < F_{OXLIM}$, then C_{Si} exhibits a significant dependence on N_{Si}, and D_{SC} increases if $N_{SiA} > N_{SiB}$.

Therefore, for the low voltage application of sub-0.1μm CMOS technologies, a degradation in capacitance scaling is expected also to come from the increased N_{Si} used to reduce short-channel effects.

Table 1 summarises the results of this work, reporting the calculated scaling degradation factor for two devices following the scaling trends reported in [6]. The first device (A) features T_{OX} = 1.5nm, $N_{Si} = 10^{18}\text{cm}^{-3}$ for V_{DD} = 1V, while the second (B) T_{OX} = 3nm, $N_{Si} = 6 \times 10^{17}\text{cm}^{-3}$ and V_{DD} = 1.5V. Three different criteria are used for comparison, i.e. (i) scaled voltages, i.e. C_{TOT} is evaluated at V_G = 1 and 1.5V for devices A and B, respectively; (ii) F_{OX} = 5MV/cm and $F_{OX} = F_{OXM}$ for metal and polysilicongates, respectively (F_{OXM} = 2 and 2.4MV/cm for devices B and A, respectively, when $N_G = 10^{19}\text{cm}^{-3}$, and 4.2 and 4.4MV/cm for devices B and A, respectively, when $N_G = 10^{20}\text{cm}^{-3}$); (iii) F_{OX} = 2.5MV/cm for both devices.

Table 1: Calculated scaling degradation factor

Case	Metal-gate	$N_G = 10^{20}\text{cm}^{-3}$	$N_G = 10^{19}\text{cm}^{-3}$
1	$D_{SC} = 0.20$	$D_{SC} = 0.34$	$D_{SC} = 0.65$
2	$D_{SC} = 0.21$	$D_{SC} = 0.33$	$D_{SC} = 0.68$
3	$D_{SC} = 0.35$	$D_{SC} = 0.42$	$D_{SC} = 0.64$

As can be seen, in the worst case, i.e. criterion 2 and $N_G = 10^{19}\text{cm}^{-3}$, C_{TOT} increases only by a factor of 1.32 instead of doubling as in the ideal case. In contrast, when considering metal-gates and comparing the devices with criterion 1, C_{TOT} increases by a factor of 1.8.

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Cluster validity index for fuzzy clustering

S.H. Kwon

A new cluster validation index is presented which can be used to eliminate the monotonically decreasing tendency when the number of clusters becomes very large and close to the number of data points. The limiting behaviour is described and numerical examples presented to show the effectiveness of the proposed cluster validity index.

Introduction: Cluster analysis for summarising data or finding 'natural' or 'real' substructures in the data set is used to place elements into groups or clusters suggested by a given data set $X = \{x_1, ..., x_n\} \subset \mathbf{R}^p$ which are n points in p-dimensional space. The fuzzy C-means (FCM) algorithm [1] for cluster analysis has been the dominant approach in both theoretical and practical applications of fuzzy techniques to unsupervised classification for the last two decades. In recent years, many functionals have been proposed for the validation of clusters of data produced by the FCM algorithm [2 – 5]. According to Pal and Bezdek's analysis [5], the Fukuyama-Sugeno index [3] is sensitive to both high and low values of weighting exponent m and may be unreliable because of this. They reported that the Xie-Beni index [4] provided the best response over a wide range of choices for 2–10 clusters and for weighting exponent m from 1.01 to 7. However, the Xie-Beni index v_{XB} monotonically decreases as the number of clusters c becomes very large and close to the number of data points n. In this Letter, we propose a new cluster validation index for eliminating the monotonically decreasing tendency as the number of clusters increases.

Cluster validity index: Among a class of cluster validity functionals such as the Dunn separation index [2], the Bezdek partition coefficient [1], the Fukuyama-Sugeno index [3], the Xie-Beni index and the extended FCM Xie-Beni index [4], we consider only the Xie-Beni index v_{XB} defined as

$$v_{XB}(U,V;X) = \frac{\sum_{j=1}^{n}\sum_{i=1}^{c} u_{ij}^2 \|x_j - v_i\|^2}{n\left[\min_{i\neq k}(\|v_i - v_k\|^2)\right]} \quad (1)$$

because it provides the best response. Xie and Beni stated that v_{XB} decreases monotonically when the number of clusters c is close to n. To avoid the indetermination due to the monotonicity, they recommended plotting v_{XB} against c, finding the starting point of the monotonic epoch as the maximum cluster number to be considered, and then selecting a value c minimising v_{XB}.

In clustering, we generally attempt to maximise intra-class similarity and inter-class differences. In this sense, a new cluster validity index v_K is defined as

$$v_K(U,V;X) = \frac{\sum_{j=1}^{n}\sum_{i=1}^{c} u_{ij}^2 \|x_j - v_i\|^2 + \frac{1}{c}\sum_{i=1}^{c}\|v_i - \bar{v}\|^2}{\min_{i\neq k}(\|v_i - v_k\|^2)} \quad (2)$$

where

$$\bar{v} = \frac{1}{n}\sum_{j=1}^{n} x_j$$

The first term of the numerator in eqn. 2 measures the intra-class similarity, i.e. how compact each and every class is. The more similar (compact) the classes, the smaller the first term is. It is independent of the number of data points. The second term in the numerator in eqn. 2 is an *ad hoc* punishing function, used to eliminate the decreasing tendency when the number of clusters c becomes very large and close to the number of data points n. The denominator in eqn. 2, which is the minimum distance between cluster centroids, measures the inter-class difference. A larger value of it indicates that every cluster is well-seperated. Our goal is to find the fuzzy c-partition with the smallest value of v_K. To investigate the limiting behaviour of the proposed index, we take a limit of the validity index as c approaches n.

(i) *Xie-Beni index:* $c \to n$: Since

$$\lim_{c\to n}\|x_j - v_i\|^2 = 0 \quad (3)$$

we have

$$\lim_{c\to n} v_{XB}(U,V;X) = \lim_{c\to n}\frac{\sum_{j=1}^{n}\sum_{i=1}^{c} u_{ij}^2 \|x_j - v_i\|^2}{n\left[\min_{i\neq k}(\|v_i - v_k\|^2)\right]} = 0 \quad (4)$$

From eqn. 4, we can see that the Xie-Beni index loses its ability to validate (U, V) pairs from the FCM for the large values of c.

(ii) *The proposed index:* $c \rightarrow n$: Since eqn. 3 holds for this case, we have

$$\lim_{c\rightarrow n} v_K(U,V;X) = \lim_{c\rightarrow n} \frac{\sum_{j=1}^{n}\sum_{i=1}^{c} u_{ij}^2 \|x_j - v_i\|^2 + \frac{1}{c}\sum_{i=1}^{c}\|v_i - \bar{v}\|^2}{\min_{i\neq k}(\|v_i - v_k\|^2)}$$

$$= \frac{\frac{1}{n}C_X}{\min_{i\neq k}(\|v_i - v_k\|^2)} \quad (5)$$

where C_X is the total scatter matrix of X. From eqn. 5, we can see that the proposed index keeps its ability to validate (U, V) pairs from the FCM for large values of c.

Numerical examples on cluster validity index: We consider three examples of data sets to show the effectiveness of the proposed cluster validity index. We first present a butterfly data set X_1 of 15 data points in $p = 2$ dimensions shown in Fig. 1*a* which has $c^* = 2$ as the number of preferred clusters. We then present two examples, X_2 of 16 data points in $p = 2$ dimensions shown in Fig. 1*b* and X_3 of 16 data points in $p = 2$ dimensions shown in Fig. 1*c*, which have $c^* = 3$ and $c^* = 4$ as the number of preferred clusters, respectively.

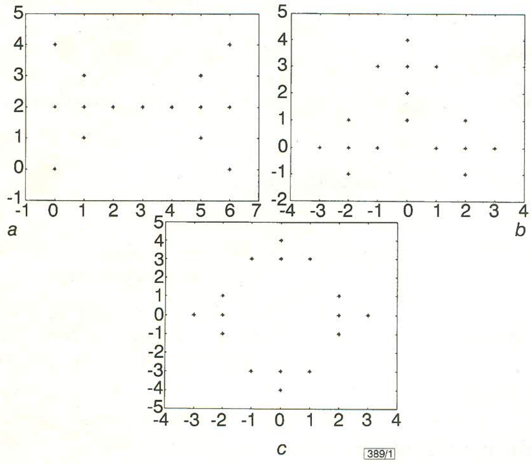

Fig. 1 *Data sets for simulation*

a Data set X_1
b Data set X_2
c Data set X_3

For each of the data sets shown in Fig. 1 we performed the FCM with the terminating criterion $\varepsilon = 10^{-8} \geq \|U_t - U_{t-1}\|$ for different weighting exponents m = 1.2, 2, 3, 4, 5, 6 and 7, and c = 2, 3, ..., n–1. Table 1 lists the value of the number of clusters chosen by each of the Xie-Beni index and the proposed index.

Table 1: c chosen by each index for data sets X_1, X_2 and X_3

m	$X_1 : c^* = 2$		$X_2 : c^* = 3$		$X_3 : c^* = 4$	
	v_{XB}	v_K	v_{XB}	v_K	v_{XB}	v_K
1.2	14	2	15	3	15	4
2.0	14	2	15	3	15	4
3.0	14	2	15	3	15	4
4.0	14	2	15	3	15	4
5.0	14	2	15	3	15	4
6.0	14	2	15	3	15	4
7.0	14	2	15	3	15	4

Since the preferred values of c are 2, 3 and 4, respectively, we see that the proposed index correctly points to the preferred values $c^* = 2$, $c^* = 3$ and $c^* = 4$ for each weighting exponent, but the Xie-Beni index points to c = 14 and 15 in each case. This behaviour is consistent with the fact that the Xie-Beni index loses its ability to validate (U, V) pairs from the FCM for large values of c, as discussed in the preceding Section. From these results, we conclude that the proposed cluster validity index shows superior performance to the Xie-Beni index, and the Xie-Beni index may be unreliable.

Conclusions: We have proposed a cluster validation index for eliminating the monotonically decreasing tendency when the number of clusters becomes very large and close to the number of data points. The limiting behaviour has been described and numerical examples showing the effectiveness of the proposed cluster validity index have been presented.

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Errata

SATO, K., HIRANO, A., ASOBE, M., and ISHII, H.: 'Chirp-compensated 40GHz semiconductor modelocked lasers integrated with chirped gratings', *Electron. Lett.*, 1998, **34**, (20), pp. 1944–1946

Editor's correction

Fig. 3 of this Letter was reproduced incorrectly. The correct Figure appears below.

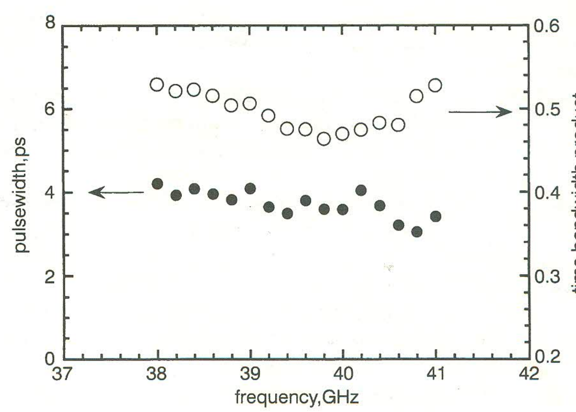

Fig. 3 *Measured pulsewidth and time-bandwidth product against modulation frequency for laser with negative type DBR*

DAVIES, B., and CONRADI, J.: 'Hybrid harmonic subcarrier optical single sideband with phase predistortion', *Electron. Lett.*, 1998, **34**, (17), pp. 1674–1675

Author's correction

In this Letter, experimental results are presented that outline a phase pre-compression method that can be used for harmonic subcarrier optical links. It is stated that the optical modulator used in the experiment consisted of a cascade of a Mach Zehnder